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(Residential Autonomous College affiliated to University of Calcutta)

B.A./B.Sc. SIXTH SEMESTER EXAMINATION, MAY 2025
THIRD YEAR (BATCH 2022-25)
MATHEMATICS (HONOURS)
Paper : CC 14

Date : 07/05/2025
Time : 11.00 am – 1.00 pm
Full Marks : 50

All the symbols have their usual meaning.

[Use a separate Answer Book for each group]

Group :A

Answer **any one** :- [1×6]

1. Show that the wave equation $a^2 u_{xx} = u_{tt}$ can be reduced to the form $u_{\xi\eta} = 0$ by the change of variables $\xi = x - at, \eta = x + at.$ Further show that $u(x,t)$ can be written as $u(x,t) = \phi(x - at) + \psi(x + at)$ where ϕ and ψ are any two arbitrary functions. [6]

2. Find the characteristics of the equation $u_{xx} - 2\sin x\, u_{xy} - \cos^2 x\, u_{yy} - \cos x\, u_y = 0.$ Write the equation in equivalent canonical form. [6]

Answer **any two** :- [2×7]

3. Deduce the D'Alembert's solution for $\frac{\partial^2 u}{\partial t^2} - c^2 \frac{\partial^2 u}{\partial x^2} = F(x,t), -\infty < x < \infty, t > 0$ subject to the initial conditions $u(x,0) = f(x), u_t(x,0) = g(x), -\infty < x < \infty.$ [7]

4. Find a formal solution of the Laplace's equation $\frac{\partial^2 \varphi}{\partial x^2} + \frac{\partial^2 \varphi}{\partial y^2} = 0$ subject to the conditions

   (i) $\varphi(0,y) = \varphi(\pi,y) = 0, 0 \le y \le \frac{\pi}{2}$ (ii) $\varphi\left(x, \frac{\pi}{2}\right) = 0, 0 \le x \le \pi$ (iii) $\varphi(x,0) = f(x), 0 \le x \le \pi$. [7]

5. Using the separation of variables, solve the following equation
   $100u_{xx} = u_t, 0 < x < 1, t > 0$ given that, $u(0,t) = 0$, $u(1,t) = 0, \ t > 0.;$ and
   $u(x,0) = \sin 2\pi x - \sin 5\pi x, 0 \le x \le 1$ [7]

Group : B

Answer **all** maximum one can score is 30 :-

6. Prove or disprove : For any contour *C*, $Re\left[\int_C f(z)\,dz\right] = \int_C Re\left[f(z)\right] dz$. [2]

7. Show that $f(z) = |z|$ is no-where differentiable. [3]

8. Find an entire function with real part $(x^3 - 3xy^2)$. [4]

9. Show that the most general Möbius transformation which maps the unit disk onto itself has the form : $z \to \lambda \frac{z - a}{\bar{a}z - 1}$ with $|a| < 1$ and $|\lambda| = 1$. [6]

10. Prove, fundamental theorem of algebra using Liouville's theorem. [5]

11. Show that for any smooth curve, $C : z(t), t \in [a,b];$ winding number of *C*, with respect to any point (which is not on *C*) is an integer. [5]

12. Find a representation for the function $f(z)=\frac{1}{1+z}$ in negative powers of z which is valid when $1<|z|<\infty$. [3]

13. Show that Taylor series of any analytic function in a given domain with respect to a given point is unique. [6]

14. State Cauchy Goursat theorem for simply connected domain and multiply connected domain. [2]

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